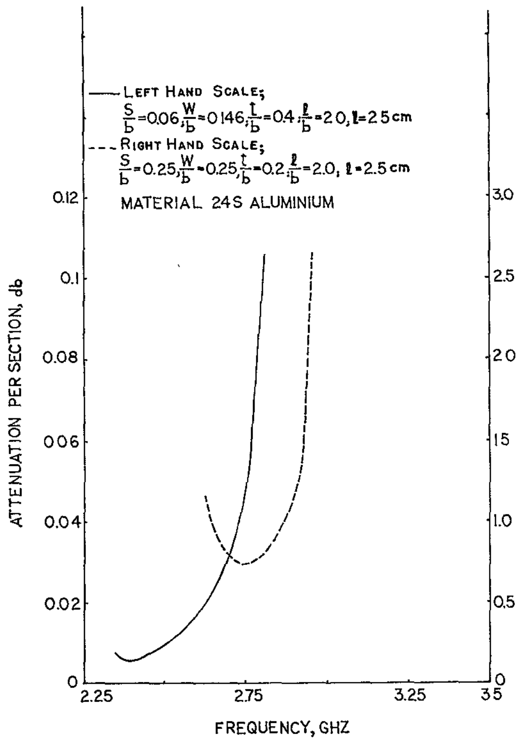

Fig. 4. Attenuation response of comb-line structures (Case 6; Table I with $C(\theta)=1$ pF).

loss identical lines the impedance parameters are found to be

$$Z_{11} = Z_{22} = Z_{33} = Z_{44}$$
$$= \tfrac{1}{2}[-j \cot \theta(Z_{oe} + Z_{oo}) + \csc^2 \theta(Z_{oe}{}^r\alpha_e l + Z_{oo}{}^r\alpha_o l)] \quad (7a)$$
$$Z_{12} = Z_{21} = Z_{34} = Z_{43}$$
$$= \tfrac{1}{2}[-j \cot \theta(Z_{oe} - Z_{oo}) + \csc^2 \theta(Z_{oe}{}^r\alpha_e l - Z_{oo}{}^r\alpha_o l)] \quad (7b)$$
$$Z_{14} = Z_{23} = Z_{41} = Z_{32}$$
$$= \frac{-j \csc \theta}{2}[(Z_{oe} + Z_{oo}) + j \cot \theta(Z_{oe}{}^r\alpha_e l + Z_{oo}{}^r\alpha_o l)] \quad (7c)$$
$$Z_{13} = Z_{31} = Z_{24} = Z_{42}$$
$$= \frac{-j \csc \theta}{2}[Z_{oe} - Z_{oo}) + j \cot \theta(Z_{oe}{}^r\alpha_e l - Z_{oo}{}^r\alpha_o l)] \quad (7d)$$

where $\theta = \beta l$. The admittance parameters are determined in a similar manner utilizing voltage sources.

Losses in a given structure may then be calculated in terms of loss due to individual sections as a function of frequency. Table I shows the expressions for attenuation per section for some typical unit filter sections consisting of coupled lines with various boundary conditions existing at individual ports of the structures.

The conductor loss per section as a function of frequency of some useful slow wave structures (filters) is plotted in Figs. 2, 3, and 4 for some typical values of structure dimensions. These are calculated utilizing the expressions given in Table I and the graphs for C_{fe}/ϵ', C_{fo}/ϵ', and C_{f}/ϵ' obtained by Getsinger [4] and Gupta [5] for coupled rectangular bars.

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Computer-Aided Renormalized Perturbation Method for the Inhomogeneously Loaded Waveguide

TAKEHIKO HIDAKA

***Abstract*—The renormalized perturbation method is applied to the inhomogeneously loaded waveguide. The second-order term in the usual Rayleigh–Schrödinger perturbation method is divergent with respect to increasing the concerned mode number. Introducing the phenomenological parameter by analogy to the quantum electrodynamics, we have the nondivergent second-order perturbation term.**

The Rayleigh–Ritz variational method and the finite-difference solution method are adequate to the eigenvalue problems of the inhomogeneously loaded waveguide, as shown in Fig. 1 [1]–[4]. Consumed computing time should be proportional to N^3 for the solution of the $N \times N$ matrix in these procedures (N is the concerned mode number in the R–R method, and in the finite-difference method, it is the total number of mesh points [4]). Since the computing time accompanying the large N is very long, we may request a more simple method for it.

The perturbation method has been believed to be applicable only to problems that are very similar to exactly solvable problems [1]. The usual Rayleigh–Schrödinger perturbation method includes some difficulties. The greatest difficulty is that the second-order (and/or higher order) terms are divergent with respect to increasing the concerned mode number, even if the loading is weak [7]. In this correspondence the extensive perturbation method, which excludes the divergence, will be given.

The Rayleigh–Schrödinger perturbation equation is described as [5], [6]

$$\gamma_i = \gamma_i{}^0 + \Gamma_{ii} + \sum_{j \neq i}^{\infty} \frac{\Gamma_{ij} \cdot \Gamma_{ji}}{\gamma_i{}^0 - \gamma_j{}^0} + \cdots \quad (1)$$

where $\gamma_i{}^0$ and $\gamma_j{}^0$ are the propagation constants of the i and j mode of the unperturbed waveguide, respectively, and Γ_{ij} is the perturbation Hamiltonian

$$\Gamma_{ij} = \langle \Psi_i{}^0 | L | \Psi_j{}^0 \rangle \quad (2)$$

where $|\Psi_i{}^0\rangle$ shows the normalized unperturbed eigenfunction of the i mode, and L is the perturbation operator [5], [6].

We discuss a thin resistive film loaded waveguide as shown in Fig. 1. In Fig. 1, if $1/R \cdot \sqrt{(\mu_0/\epsilon_0)} \cdot (d/b)$ is constant with respect to the change of d/b, then the first-order perturbation term in (1) is constant, and thus we call it the constant loading. Physically, it is expected that, according to $d/b \rightarrow 0$ (i.e., $R \rightarrow 0$) under the constant loading, the perturbed propagation constant γ_i goes near to $\gamma_i{}^0$, because the $d/b = 0$ load gives no perturbation for the electromagnetic wave, even if the load has high conductivity.

The result of Rayleigh–Schrödinger calculation for constant loading is shown in Fig. 1. At small d/b, i.e., small R, the calculated propagation constant is divergent. This situation is the same as the

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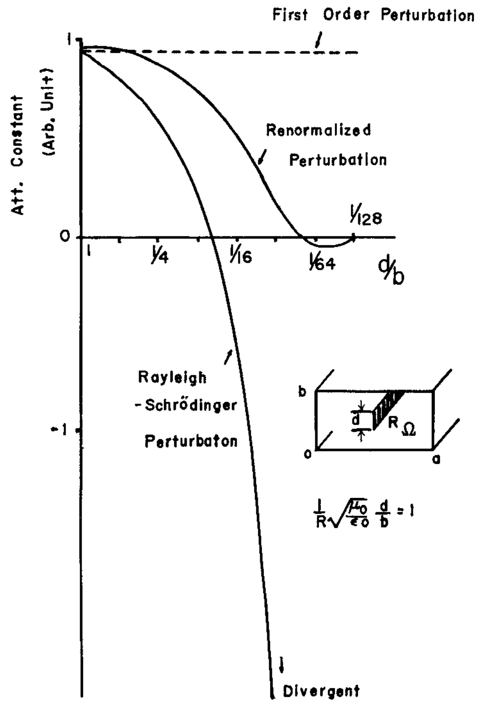

Fig. 1. Example of the renormalized perturbation method for the inhomogeneously loaded waveguide. The load $(1/R)\cdot\sqrt{(\mu_0\cdot\epsilon_0)}\cdot d\cdot b$ is 1.0, frequency $2a/\lambda_0$ is 1.4, and a/b is 2.0. The load is a thin-film resistor in the center of the waveguide.

divergence of the second-order perturbation in the quantum electrodynamics [7].

The "Renormalization Theory" is introduced by analogy with quantum mechanics to exclude the divergence. The perturbation operator $L' = L + \delta L_\alpha + \delta L_\beta$ is substituted for L in our renormalized perturbation method. δL_α and δL_β are the new operators used to cancel the real and imaginary part of the second-order term in (1)

$$\delta L_\alpha = -\operatorname{Re}\lim_{d\to 0}\sum_{j\neq i}^{\infty}\frac{\Gamma_{ij}\cdot\Gamma_{ji}}{\gamma_i^0-\gamma_j^0}$$
$$\delta L_\beta = -\operatorname{Im}\lim_{d\to 0}\sum_{j\neq i}^{\infty}\frac{\Gamma_{ij}\cdot\Gamma_{ji}}{\gamma_i^0-\gamma_j^0} \quad (3)$$

which are divergent with $d/b\to 0$ [7]. δL_α and δL_β are proportional to $1/(d/b)$ and $1/(d/b)^2$, respectively, in the region of small d/b:

$$\delta L_\alpha = A_\alpha\cdot 1/(d/b)$$
$$\delta L_\beta = A_\beta\cdot 1/(d/b)^2. \quad (4)$$

In computer-aided calculation, we cannot execute Γ_{ij} for the infinitely small d/b (at the same time infinitely small R) region and the infinite summation of the second-order term in (1). To be finite, we have to "cut off" the d/b value and the concerned mode number. For actual computer calculation, first, we assume the d_0/b value small enough, and determine the factors A_α and A_β in (4) to cancel the second-order term in (1) at this stage. For the region $d>d_0$, we postulate that these factors A_α and A_β are constant. If $d\neq d_0$, the second-order term in (1) and δL are not cancelled with each other. The residual second-order term subtracted by δL is the observed value at $d>d_0$ [7].

An example is shown in Fig. 1. The concerned mode number is 24 051, which cannot be treated by the variational method.

The load $1/R\cdot\sqrt{(\mu_0/\epsilon_0)}\cdot(d/b)$ is 1.0. The cutoff value d_0/b for computer-aided calculation is 1/128 ($R\approx 3\Omega^{\square}$). The Rayleigh–Schrödinger method shows the strong divergence, whereas our renormalized perturbation method gives physically reasonable values. The calculation time for our new method is proportional to N, which is smaller than for the variational or finite difference methods.

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Gunn-Effect Power Limiter

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***Abstract*—The possibility of using the nonlinearity of the Gunn device current–voltage characteristic to provide microwave power limiting is discussed. Initial pulsed and CW results are presented which demonstrate limiting action.**

I. Introduction

The use of the Gunn effect in an oscillator is well known. Little attempt has been made to exploit the nonlinear nature of the Gunn effect mechanism for other applications such as mixers, harmonic generators, parametric amplifiers, and limiters.

Sterzer [1] has used the Gunn effect to construct an amplitude modulator. Aitchison [2] and others have observed parametric amplification.

This correspondence suggests that the Gunn-effect device could be used as a microwave power limiter.

II. Discussion

It is known that the Gunn-effect dc current–voltage characteristic is nonlinear and, while varying from sample to sample, is often of the form shown in Fig. 1; only some of the apparent nonlinearity is due to heating.

Work at microwave frequencies at Mullard Research Laboratories has demonstrated that a similar characteristic is available up to 3 GHz and it will be assumed in this discussion that this characteristic exists beyond 3 GHz.

The application of a microwave signal to a Gunn sample mounted in shunt with a transmission line of suitable impedance will produce a voltage across the Gunn sample; the current which flows will be a function of the applied bias and the magnitude of the voltage. A Fourier analysis of the current will show a mean (dc) component plus a component at the incident frequency (plus other components at higher frequencies which are neglected). Both the mean component and the incident frequency component will be functions of power level and the detailed behavior will be a function of the low signal bias.

In particular, the small signal conductance at the turnover point will be zero and using an established simple equivalent circuit of the Gunn sample in the form of a shunt capacitance and a shunt conductance we may expect a Gunn sample placed across a transmission line to have a small insertion loss when the appropriate bias is applied.

Increase in applied power at the incident frequency will have two effects—both of which will introduce a finite insertion loss thereby giving a limiting action. The mean value of conductance will change,

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